OPTIMIZATION OF PRISM SURFACE SHAPE IN INTERACTION WITH FLUID FLOW

Kristaps Spade[1], Janis Viba[1], Martins Irbe[1], Shravan Koundinya Vutukuru[1, 2]
[1]Riga Technical University, Latvia; [2]MLR Institute of Technology, India
kristaps.spade@rtu.lv, janis.viba@rtu.lv, martins.irbe@rtu.lv, vshravankoundinya1989@gmail.com

Abstract. The interaction of the lateral surface of an axially cylindrical prism with the fluid flow is studied. The prism moves in a translational motion (without rotation) in a fixed fluid, such as air. The effectiveness of such a reduced interaction analysis is important in reducing vehicle drag as well as increasing the speed of technical sports vehicles. Accordingly, the task of increasing the interaction forces involves extracting energy from the fluid. In the mathematical problem, the force of resistance of the prism to fluid movement, its minimum or maximum value, is chosen as the optimization criterion. The interaction between the prism and the fluid is described in an unconventional way, without using the concepts of drag and lift forces, but using the relationships of classical mechanics. For this purpose, the interaction of the prism with the fluid is divided into two zones: the pressure zone (in front of the prism) and the suction zone (at the back of the prism). Interactions with changes in the amount of motion of a fluid in a differential form are obtained. The relationships found are integrated in the simplest cases: for example, when the surfaces of a prism are broken planes. Two dominant forms of the prism are considered: the surfaces are only convex or the surfaces are only concave. Parametric optimization problems are solved numerically with a computer. As a result, optimal shapes are obtained for the minimum criterion and the maximum criterion. The main result of the research is the application of a new theory of fluid mechanics, which allows analytical or numerical solving of analysis, optimization and synthesis problems with the obtained formulas, without the use of space time programming, which would need to change the object shape, flow rate and direction in almost every integration step.

Keywords: fluid flow, optimization, concave, convex plane.

Introduction

The application of vibration motion to human activities in the environment is very common and well-studied. Fluid flow is widely used to excite vibrations [1-3]. The most important indicator in the calculation of the fluid flow interaction is the force acting on the object. Experimental drag andlLift coefficients are mostly used in existing force calculation methods [4-6]. Another approach is used in this work, when the force on the frontal pressure side is determined by the laws of classical mechanics, which can be traced back to Newton’s work [7-14]. In addition, the suction on the outlet side is observed with one constant C, which has a very narrow range of values, bounding around C = (0.25-0.50) [15-18]. This approach makes it possible to obtain analytical interaction force formulas that contain the geometrical parameters of the prism, such as the face length and the face position angle. As a result, a methodology for optimizing the interaction force is developed and the optimization results for a curved and concave prism are given. In addition, the obtained results are illustrated by modelling a continuous environment with a computer Ansys Fluent Pressure-Based Steady state 2D flow analysis. It is shown that by changing the shape of the prism, it is possible to induce fluctuations in the constant fluid flow, which can be used for energy production. The present work shows the advantages of approximate analytical formulas in shape optimization when fluid interacts with a broken plane. To solve such a problem with a standard fluid dynamics program would require a large consumption of resources in a two-dimensional space, changing the lengths and angles when calculating the criterion. Instead, the surface of the response, calculated analytically with the found force formula, is shown. As expected, the optimal solutions lie on the constraints of a parameter. In general, it should be noted that the numerical methods of fluid motion and interaction with objects require large material resources, as the results depend not only on the shape of the objects, but also on the rules of the onset of fluid motion. In addition, there are problems in comparing the obtained numerical results with experimental tests in suction wind tunnels, which do not correspond to the natural process.

Analytical 2D convex and concave plane body interaction with fluid flow model

The two-dimensional shape is described in the following Fig. 1. Consider a prism symmetric to the flow rate $V0$ with a side width B that is constant. Accordingly, the width L and the height H of the prism are also constant.

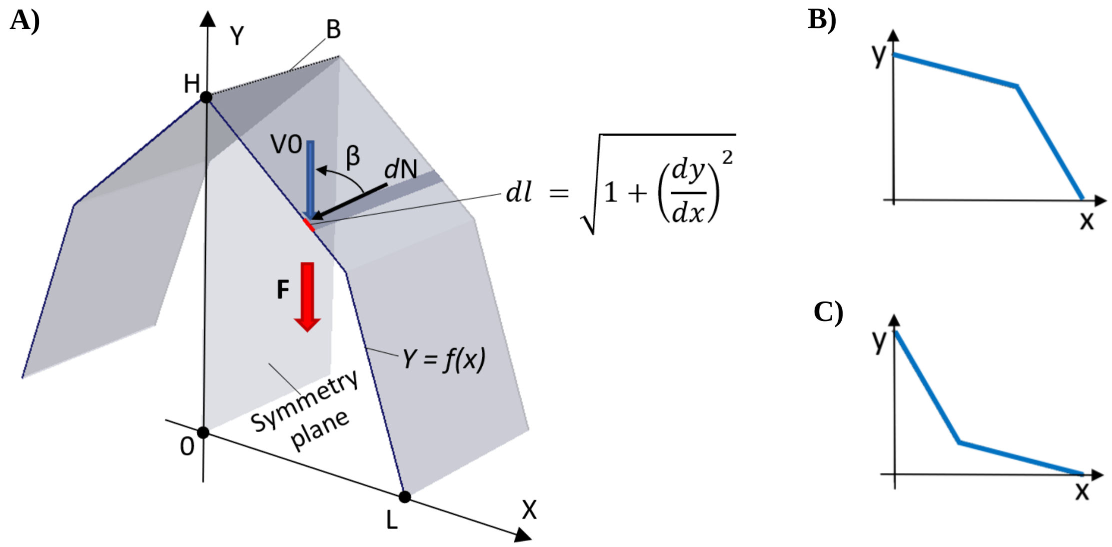

Fig. 1. **Two dimensional (x-y plane) curved line model (A); Two convex planes (B); Two concave planes (C):** *V*0 – fluid flow velocity before obstacle; L – half the width of the prism; *F* – force of frontal interaction; *H* – height of the prism; *B* – side length of the prism

Optimization problem of given prism with the length B and shape characterising function *f*(*x*) can be formulated as follows:

1. The boundary conditions for given shape of model are $y = f(x)$; $y\,(x = 0) = H$; and at $y\,(x = L) = 0$. The task is to find the optimal shape of $f(x)$ that provides the minimum flow force F if the feed rate is V0. The importance of the task increases if it is possible to obtain energy savings;
2. And task is similar for the second case where is also taken up the criterion: to find the shape of the model $f(x)$ that provides the maximum flow force F. In this case, the task would be important in energy production systems.

To solve the shape optimization task from Fig. 1 the following analytical relationships (1) and (2) are obtained:

$$\begin{aligned} &dN = dl \cdot B \cdot [V0 \cdot \cos(\beta)]^2, \\ &dl = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}, \\ &\beta = \operatorname{atan}\left(\frac{dy}{dx}\right), \\ &dF = dN \cdot \cos(\beta), \\ &F = \int_{x,y} dF, \end{aligned} \tag{1}$$

$$\delta F = 0 \text{ for Supermum; infimum,} \tag{2}$$

where dN, dF – components of the local normal force and local axial force, N;
dl – length of local arc, m;
δF – variation of force F inside limits or on the boundary limits (in case Supremum, infimum).

One way to solve such a shape optimization problem is to choose a polynomial shape as follows (3):

$$y = H + C_1 \cdot x + C_2 \cdot x^2 + C_3 \cdot x^3 + \ldots + \ldots, \tag{3}$$

where C_1, C_2, C_3 – constants.

The main disadvantage of this method is the large computational capacity, finding the polynomial constants and forming a real shape from them, which will probably depend on the flow rate V0. Therefore, in our work, let us look at another simpler form, which consists of two planes, as an example (Fig.1. A, B).

First Task – model of two convex planes, Fig 1. A.

Consider the form that differs from the shape of a polynomial in that it is formed by two straight lines (prism side planes) with $L = H$, Fig. 2.

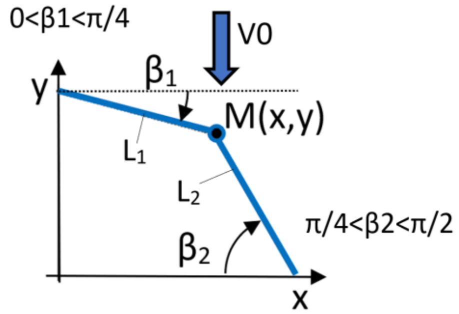

Fig. 2. **Two convex plane model *L = H*:** L_1, β_1 – parameters of the first plane; L_2, β_2 – parameters of the second plane

In the next step it is needed to develop a mathematical relation for each of the lengths L_1 and L_2.

According to the authors' new theory of calculating the interaction force F for a flat convex broken plate, the following formula should be used (4) [18]:

$$F = (1 + C) \cdot V0^2 \cdot \rho \cdot B \cdot L \cdot D. \tag{4}$$

Here

$$D = \frac{l_1 \cdot [\cos \beta_1]^3 + l_2 \cdot [\cos(\beta_2)]^3}{L}, \tag{5}$$

where C – constant, approximately 0.5;
$V0$ – flow velocity;
ρ – density of flow;
l_1, l_2, L – parameters of broken plate;
β_1, β_2 – orientation angles of plate parts.

It can be deduced from expressions (4), (5) that the variable part of the interaction force F is a function D (5). Therefore, when optimizing this function, the following relationships between the lengths l_1, l_2 and the angles β_1, β_2 (6) must be observed (6):

$$\begin{aligned} L &= L_1 \cdot \cos(\beta_1) + L_2 \cdot \cos(\beta_2), \\ H &= L_1 \cdot \sin(\beta_1) + L_2 \cdot \sin(\beta_2). \end{aligned} \tag{6}$$

From expressions (4)-(6) it can be concluded that the optimization problem must be solved numerically. Here we illustrate the case when $L = H = 0.2$ m. Then the optimization criterion D takes the following expression (7):

$$D = \frac{\sin(\beta_2) \cdot \cos(\beta_1)^3 - 1.0 \cdot \cos(\beta_1)^3 \cdot \cos(\beta_2) + \cos(\beta_1) \cdot \cos(\beta_2)^3 - 1.0 \cdot \sin(\beta_1) \cdot \sin(\beta_2)^3}{\cos(\beta_1) \cdot \sin(\beta_2) - 1.0 \cdot \cos(\beta_2) \cdot \sin(\beta_1)}. \tag{7}$$

The results of the optimization problem from expression (7), when the angles β_1, β_2 change, are graphically shown in Fig. 3. Here, the values of the coefficient D are plotted as the surface of the criterion in three-dimensional space. Conclusions about higher and lower criterion values are given in the description of the drawing, Fig. 3.

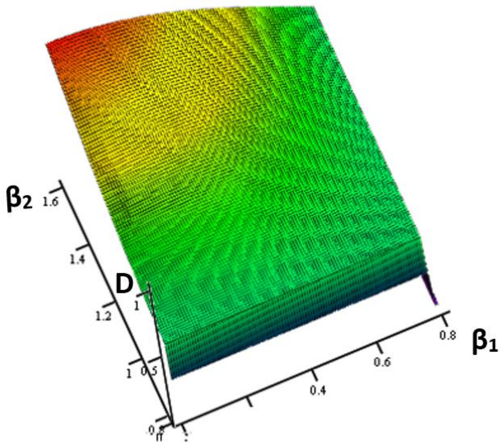

Fig. 3. **Maximum and minimum drag force depending on convex plane angle**
F_{max} is at $\beta_1 = 0$; $\beta_2 = \pi/2$ (90°). F_{min} is at $\beta_1 = \beta_2 = \pi/4$ (45°)

Second Task – model of two concave planes, Fig 1. B.

In the case of a concave prism surface, the criterion $D_1(x, y)$ was expressed as a function of the coordinates x and y of the breaking point. The resulting expression is quite long and is not inserted here. Numerical optimization results D_1 at similar parameters $L = H = 0.2$ m are shown in Fig. 4.

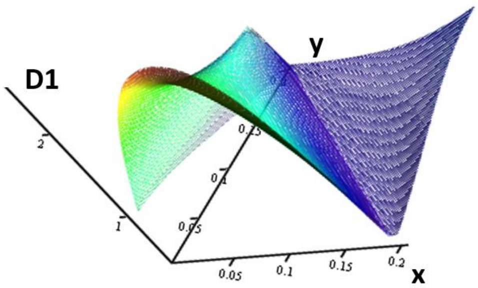

Fig. 4. **Maximum and minimum drag force depending on concave plane angle**
F_{max} is at region, $y = 0$ and $0 < x < L = 0.2$; The minimum force is at $\beta_1 = \beta_2 = \pi/4$ (45°) or $x = y$

Numerical fluid flow calculations by Ansys Fluent

In order to understand and evaluate the theoretically obtained optimization results, the interaction modelling was performed with Ansys Fluent. Pressure-Based Steady state 2D flow analysis is done with k-ω SST viscous model. The air is the fluid with constant density $\rho = 1.225$ kg·m^{-3}. The length of planes $L_1 = L_2 = 1$ m. For modelling five geometries are created, Fig. 5, where for models 2-3 the condition $L = H$ is retained.

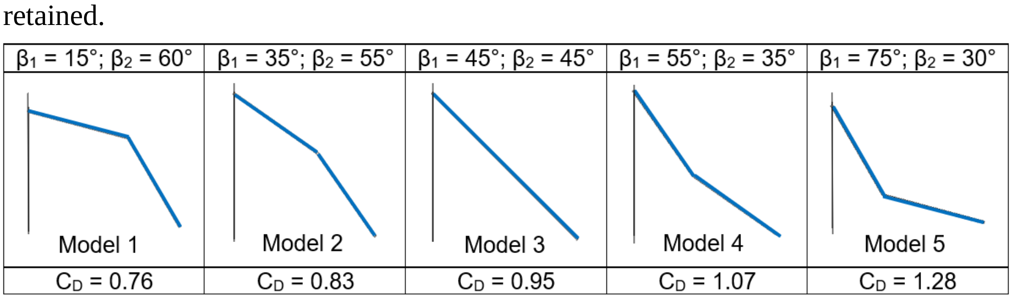

Fig. 5. **2D model shapes and detected drag coefficient C_D**

Simulation of the pressure on the 2D body in Fig. 6 showed that the detected drag coefficient for models of concave planes and curved planes changes significantly.

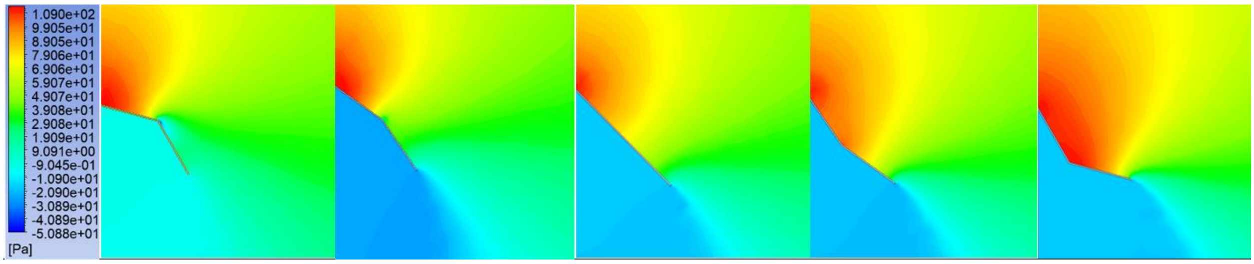

Fig. 6 **Pressure distribution for 2D model shapes**

In Fig. 7. in detail is showed pressure distribution on the frontal face of planes.

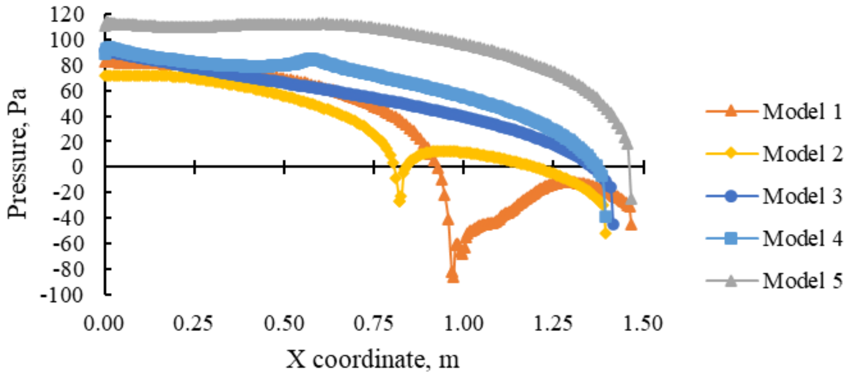

Fig. 7. **Frontal pressure distribution of 2D models shapes**

Results and discussion

1. It is not appropriate to use drag and lift coefficients to describe the fluid flow interaction, as these must be determined experimentally in advance.
2. It is desirable to describe the interaction between a fluid and an object with a change in the amount of fluid motion in a differential form [16-18]. This results in the analytical relationships already described in Newton's work.
3. Calculation of parametric optimization of the prism shape of a broken, flat plate has been performed. The calculation shows which shapes are optimal, depending on the chosen criterion: whether the minimum or maximum force should be obtained.
4. With computer modelling "Fluent" qualitatively confirmed the theoretical studies, which show that the force of interaction in concave prisms increases significantly, even 1.8 times. This fact is used in energy production systems.

Conclusions

1. The main result of the research is the application of a new theory of fluid mechanics, which allows analytical or numerical solving of analysis, optimization and synthesis problems with the obtained formulas.
2. Optimization of the prism shape is performed in the work, which gives information in the synthesis of new shapes that can be used in the synthesis of real mechanisms with mechatronic control, for example, to switch from one shape to another to obtain energy from oscillating flow.
3. The work shows that the cavities of the object significantly increase the interaction forces approximately doubling them. In the calculations, the cavities must not be approximated by flat covers based on the present research.

Author contributions:

Indicate the contribution of each author. Conceptualization, VJ.V.; methodology, S.V. and M.I.; software, M.I; validation, K.S. and M.I.; formal analysis, J.V. and S.V.; writing – review and editing, K.S. and M.I.; visualization, M.I.; All authors have read and agreed to the published version of the manuscript.

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